There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry. **SCORE: ______ / 7 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **You do NOT need to prove the identity.**

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

[b] Write the identity for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$ simultaneously. **You do NOT need to prove the identity.**

$$\cosh 2x = \cosh^2 x + \sinh^2 x \quad (1)$$

[c] Use the results of [a] and [b] to find and **prove** an identity for $\cosh 2x$ that uses only $\sinh x$.

$$\cosh 2x = (1 + \sinh^2 x) + \sinh^2 x \quad (1)$$

$$= 1 + 2\sinh^2 x \quad (\tfrac{1}{2})$$

[d] If $\tanh x = -\frac{2}{5}$, find $\sinh x$ using identities.

**You must explicitly show the use of the identities but you do NOT need to prove the identities.**

**Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.**

$$\operatorname{sech}^2 x = 1 - \tanh^2 x$$

$$= 1 - \frac{4}{25} \quad (1)$$

$$= \frac{21}{25}$$

$$\operatorname{sech} x = \frac{\sqrt{21}}{5} \quad (\tfrac{1}{2})$$

SINCE $\operatorname{sech} x > 0$ FOR ALL $x \in \mathbb{R}$

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{1}{\frac{\sqrt{21}}{5}} = \frac{5}{\sqrt{21}} \quad (\tfrac{1}{2})$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

so $\sinh x = \tanh x \cosh x$

$$= -\frac{2}{5} \cdot \frac{5}{\sqrt{21}} \quad (1)$$

$$= -\frac{2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21} \quad (\tfrac{1}{2})$$

Rewrite $\operatorname{csch}\left(\frac{1}{2}\ln 3\right)$ in terms of exponential functions and simplify. **SCORE: ______ / 3 PTS**

$$\frac{2}{e^{\frac{1}{2}\ln 3}-e^{-\frac{1}{2}\ln 3}} = \frac{2}{e^{\ln 3^{\frac{1}{2}}}-e^{\ln 3^{-\frac{1}{2}}}} = \frac{2}{\sqrt{3}-\frac{1}{\sqrt{3}}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3-1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

(1) (1) ($\frac{1}{2}$) ($\frac{1}{2}$)

Write and **prove** a formula for $\cosh(x-y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: ______ / 6 PTS**

$$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y \quad (1)$$

$$= \frac{e^x+e^{-x}}{2}\cdot\frac{e^y+e^{-y}}{2} - \frac{e^x-e^{-x}}{2}\cdot\frac{e^y-e^{-y}}{2} \quad (1)$$

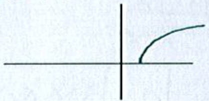
$$= \frac{(e^{x+y}+e^{x-y}+e^{-x+y}+e^{-(x-y)})-(e^{x+y}-e^{x-y}-e^{-x+y}+e^{-(x-y)})}{4} \quad (1)\ (1)\ (\tfrac{1}{2})$$

$$= \frac{2e^{x-y}+2e^{-x+y}}{4} \quad (1)$$

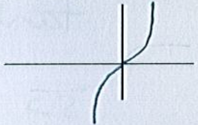
$$= \frac{e^{x-y}+e^{-(x-y)}}{2} \quad (\tfrac{1}{2})$$

Sketch the general shape and position of the following graphs. **SCORE: ______ / 3 PTS**
(Don't worry about specific $x-$ or $y-$ coordinates.) GRADED BY ME

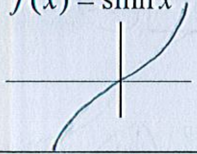
$f(x) = \cosh^{-1} x$



$f(x) = \tanh^{-1} x$



$f(x) = \sinh x$



Prove that $g(x)=\frac{1}{2}\ln\frac{1+x}{1-x}$ is the inverse of $f(x)=\tanh x$ by simplifying $f(g(x))$. **SCORE: ______ / 5 PTS**

**You may use any identities that you found in part [1] of the Hyperbolic Functions Supplement without proving them.**

$\tanh x\left(\frac{1}{2}\ln\frac{1+x}{1-x}\right)$

$= \dfrac{e^{\frac{1}{2}\ln\frac{1+x}{1-x}} - e^{-\frac{1}{2}\ln\frac{1+x}{1-x}}}{e^{\frac{1}{2}\ln\frac{1+x}{1-x}} + e^{-\frac{1}{2}\ln\frac{1+x}{1-x}}}$ (1)

$= \dfrac{\sqrt{\frac{1+x}{1-x}} - \frac{1}{\sqrt{\frac{1+x}{1-x}}}}{\sqrt{\frac{1+x}{1-x}} + \frac{1}{\sqrt{\frac{1+x}{1-x}}}} \cdot \dfrac{\sqrt{\frac{1+x}{1-x}}}{\sqrt{\frac{1+x}{1-x}}}$ (1)

$= \dfrac{\frac{1+x}{1-x} - 1}{\frac{1+x}{1-x} + 1} \cdot \dfrac{1-x}{1-x}$ (1)

$= \dfrac{1+x-(1-x)}{1+x+(1-x)}$ (1)

$= \dfrac{2x}{2}$

$= x$ (1)